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12. **Proposed by H. W. DRAUGHON, Clinton, Louisiana.**

Find three numbers such that, the sum of their cubes may be a square, and the sum of their squares a cube.

Solution by ARTEMAS MARTIN, LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let ax, bx and cx denote the numbers; then $(a^3+b^3+c^3)x^3=\square \ldots\ldots(1)$, $(a^2+b^2+c^2)x^2=\text{cube},=x^3$ say$\ldots\ldots(2)$ and we have $x=a^2+b^2+c^2$.

Substituting in (1), after expunging x^2, $(a^2+b^2+c^2)(a^3+b^3+c^3)=\square \ldots\ldots(3)$.

Let $mv=a$, $nv=b$, $pv=c$; then (3) becomes

$$(m^2+n^2+p^2)(m^3+n^3+p^3)v=\square,=v^2 \text{ say, after rejecting } v^4\text{; whence}$$

$$v=(m^2+n^2+p^2)(m^3+n^3+p^3).$$

$$\therefore\quad a=m(m^2+n^2+p^2)(m^3+n^3+p^3),$$
$$b=n(m^2+n^2+p^2)(m^3+n^3+p^3),$$
$$c=p(m^2+n^2+p^2)(m^3+n^3+p^3);$$

$$\text{and } x=a^2+b^2+c^2=(m^2+n^2+p^2)^3(m^3+n^3+p^3)^2.$$

$$\text{Hence } ax=m(m^2+n^2+p^2)^4(m^3+n^3+p^3)^3,$$
$$bx=n(m^2+n^2+p^2)^4(m^3+n^3+p^3)^3,$$
$$cx=p(m^2+n^2+p^2)^4(m^3+n^3+p^3)^3.$$

If $m=1$, $n=2$, $p=3$, the numbers are, after dividing out the 6th power factor 6^6, 38416, 76832 and 115248.

Also solved by *H. W. DRAUGHON, F. P. MATZ, and G. B. M. ZERR.*

PROBLEMS.

18. **Proposed by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.**

Decompose into the sum of two squares the number $17^3 . 73^5$.

19. **Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Find three positive integer numbers whose sum is a cube, and, also, the sum of any two diminished by the third a cube.

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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

5. **Proposed by DE VOLSON WOOD, M. A., C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

An actual case suggested the following:

An equal number of white and black balls of equal size are thrown into a rec-

angular box, what is the probability that there will be contiguous contact of white balls from one end of the box to the opposite end? As a special example, suppose there are 30 balls in the length of the box, 10 in the width, and 5 (or 10) layers deep.

Solution by Professor P. H. PHILBRICK, M Sc., C. E., Lake Charles, Louisiana.

The least number of balls forming a row is 30; and the greatest number is 30+9+4=43.

Suppose the box to be divided into $30\times10\times5=1500$ equal cubes an end of the box being shown in the figure. Then, in general, a row may connect any cube at one end with any one of a determinate number of cubes at the other end of the box. Call the diameter of a ball unity then the latteral deviation of the rows may range from 0 to 9 and the vertical deviation from 0 to 4.

Let m=the number of rows in the width =10 and n=the number of rows in the depth=5. Then the following table shows the number of rows of given numbers of balls in a row so far as the beginning and the ending of the rows are concerned. The figures over the numbers of rows show the deviations of the rows latterally and vertically or vertically and latterally.

30 mn $=50$

31 $2(m-1)n+2m(n-1)=90+80=\overset{1,0}{170}$ $=170$

32 $2(m-2)n+2m(n-2)+4(m-1)(n-1)=\overset{2.0}{140}+\overset{1,1}{144}$ $=284$

33 $2(m-3)n+2m(n-3)+4(m-2)(n-1)+4(m-1)(n-2)=\overset{3,0}{110}+\overset{2.1}{236}$ $=346$

34 $2(m-4)n+2m(n-4)+4(m-3)(n-1)+4(m-1)(n-3)+4(m-2)(n-2)$
$=\overset{4.0}{80}+\overset{3,1}{184}+\overset{2.2}{96}=360$

35 $2(m-5)n+4(m-4)(n-1)+4(m-1)(n-4)+4(m-3)(n-2)+4(m-2)(n-3)$
$=\overset{5,0}{50}+\overset{4,1}{32}+\overset{3,2}{148}=330$

36 $2(m-6)n+4(m-5)(n-1)+4(m-4)(n-2)+4(m-2)(n-4)+4(m-3)(n-3)$
$=\overset{6,0}{40}+\overset{5.1}{80}+\overset{4,2}{104}+\overset{3,3}{56}=280$

37 $2(m-7)n+4(m-6)(n-1)+4(m-5)(n-2)+4(m-4)(n-3)+4(m-3)(n-4)$
$=\overset{7,0}{70}+\overset{6,1}{64}+\overset{5,2}{60}+\overset{4,3}{76}=230$

38 $2(m-8)n+4(m-7)(n-1)+4(m-6)(n-2)+4(m-5)(n-3)+4(m-4)(n-4)$
$=\overset{8,0}{20}+\overset{7,1}{48}+\overset{6.2}{48}+\overset{5,3}{40}+\overset{4.4}{24}=180$

39 $2(m-9)n+4(m-8)(n-1)+4(m-7)(n-2)+4(m-6)(n-3)+4(m-5)(n-4)$
$=\overset{9,0}{10}+\overset{8,1}{32}+\overset{7.2}{36}+\overset{6.3}{32}+\overset{5,4}{20}=130$

40 $4(m-9)(n-1)+4(m-8)(n-2)+4(m-7)(n-3)+4(m-6)(n-4)$
$=\overset{9,1}{16}+\overset{8.2}{24}+\overset{7,3}{24}+\overset{6,4}{16}=$ 80

41 $4(m-9)(n-2)+4(m-8)(n-3)+4(m-7)(n-4)=\overset{9,2}{12}+\overset{8,3}{16}+\overset{7.1}{12}$ $=40$

42 $4(m-9)(n-3)+4(m-8)(n-4)=\overset{9,3}{8}+\overset{9.3}{8}$ $=16$

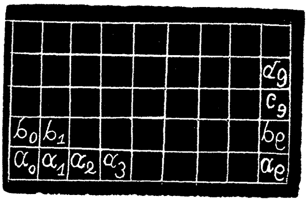
43 $4(m-9)(n-4)=\overset{9,4}{4}$ $=4$

The law of formation is apparent.

But each row may zigzag in many ways and occupy many positions. Let the figure, for the present, represent any layer of balls and let us consider the ways of passing from a_0 to b_9, c_9, etc.

To pass to b_9 we may jog upward at a_0 a_1 .... or a_9 giving 10 ways.

To pass to c_9 if we jog upward one space at a_0 (to b_0), then as just shown there are 10 ways to c_9; if the jog is at a_1 (to b_1) there are 9 ways to c_9; if the jog is at a_2 (to b_2) there are 8 ways to c_9, etc. Hence from a_0 to c_9 there are $10\times 9+\ldots\ldots+1=\frac{11\times 10}{1.2}=C_2^{11}$ ways. To pass to d_9 if we jog from a_0 to b_0 then from b_0 to d_9 is the same as from a_0 to c_9 or as just shown, C_2^{11} ways.



From b_1 to d_9 there are for same reason $9+8+\ldots\ldots+1=\frac{10+9}{1.2}=C_2^{10}$ ways. etc., etc.

Hence in all there are

$$C_2^{11}+C_2^{10}+\ldots\ldots+C_2^{2}=C_3^{12}=C_9^{12} \text{ ways.}$$

Hence, in general, if $m=$ the sum of the number of spaces or steps in both directions between the centers of the first and the last ball, and $n=$ the number in either direction, the number of ways from the first to the last $=C_n^m$. Observe that the number of balls $=m+1$.

Now consider rows that begin and end in different layers.

Let $N=$ the number of balls in any row. Consider two layers. Suppose *any* row beginning and ending at given points in adjacent horizontal layers to be projected on the lower layer.

There are $N-1$ balls and $N-2$ spaces in the projected row; and hence $N-2-29=N-31$ spaces latterally. Hence the number of projected rows is C_{N-31}^{N-2}. It is necessary to find the number of ways each projected row (and therefore the original row) can be divided between the two layers. To do this we observe that the real length of the row is the length of the box added to the latteral deviation of the row.

Since there are $N-2$ spaces horizontally and one space vertically, the number of ways of distributing the N balls of any projected row on the two layers is, $C_1^{N-2+1}=C_1^{N-1}$.

Hence any two given points on opposite ends of two adjacent layers can be connected with N ball in $C_1^{N-1}.\,C_{N-31}^{N-2}$ ways.

$$\text{Now } C_1^{N-1}.\,C_{N-31}^{N-2}=\frac{N-1}{1}.\frac{(N-2)(N-3)\ldots\ldots 30}{1.\,2.\,3\ldots\ldots(N-31)}.$$

$$\text{But } C_{N-30}^{N-1}=\frac{(N-1)(N-2)\ldots\ldots 30}{1.2.3\ldots\ldots(N-31)(N-30)}.$$

$$\text{Hence } C_1^{N-1}.\,C_{N-31}^{N-2}=N-30\,C_{N-30}^{N-1}=C_1^{N-30}.\,C_{N-30}^{N-1} \text{ ways.}$$

Let there be three layers.

Suppose any row to be projected upon the lower layer. There are $N-2$ balls and $N-3$ spaces horizontally. Also two spaces vertically and $N-3-29=N-32$ latterally.

Hence the total number of projected rows is, C^{N-3}_{N-32}. Each row can be distributed in, $C_2^{N-3+2}=C_2^{N-1}$ ways. Hence any two given points on opposite ends of the first and last of three adjacent layers can be connected with N balls in $C_2^{N-1}.C^{N-3}_{N-32}$ ways.

$$\text{Now } C_2^{N-1}.C^{N-3}_{N-32}=\frac{(N-1)(N-2)}{1.2}\times\frac{(N-3)(N-4)\dots.30}{1.2.3.\dots.N-32}$$

$$\text{and } C^{N-1}_{N-30}=\frac{(N-1)(N-2)(N-3)(N-4)\dots.30}{1.2.3.\dots.(N-32)(N-31)(N-30)}.$$

$$\therefore\quad C_2^{N-1}.\ C^{N-3}_{N-32}=\frac{(N-30)(N--31)}{1.2}.\ C^{N-1}_{N-30}=C_2^{N-30}.C^{N-1}_{N-30}\text{ ways.}$$

Similarly for four adjacent layers there are, $C_3^{N-30}.C^{N-1}_{N-30}$ ways,

and for five adjacent layers, $C_4^{N-30}.C^{N-1}_{N-30}$ ways.

For n adjacent layers there are, $C^{N-30}_{N-1}.C^{N-1}_{N-30}$ ways.

Now out of 750 white balls 30 may be drawn in,

$$\frac{750.749\dots.721}{1.2.3\dots.30}=a\text{ ways.}$$

Also $1500-30=1470$ may be drawn in $\underline{|1470}=b$ ways.

31 white balls may be drawn in $\frac{750.748\dots.720}{1.2.3\dots.31}=a_1$ ways, and

$1500-31$ may be drawn in $\underline{|1469}=b_1$ ways. Similarly for other numbers.

Represent the numbers of ways of drawing $32,33\dots.43$ balls by a_1, $a_3\dots.a_{13}$ and the ways of drawing the complementary numbers $1468,1467\dots.1457$ by b_2, $b_3\dots.b_{13}$.

Now we have as follows:

30 white balls may be drawn in a ways, 1470 in b ways. There are of 30 balls each 50 rows, and each row has $C_0^{29}=1$ position.

Hence such a row may be formed in $50\,C_0^{29}\,ab=50\,ab$ ways.

Since there are 170 rows of 31 ball each and each row may take C_1^{30} positions such a row may be formed in $170\ C_1^{30}\ a_1\ b_1=5100\ a_1\ b_1$ ways.

There are 120 rows of 32 balls each in the same layer, and 144 rows oocupying adjacent layers, each one of the first may take C_2^{31} positions and each one of the second set $2\ C_2^{31}$ positions.

Hence such a row may be formed in $(140+2.144)\,C_2^{31}a_2b_2=428\,C_2^{31}a_2b_2$ ways.

There are 110 rows of 33 balls each in the same layer and 236 occupying adjacent layers.

Hence such a row may be formed in $(110+3.236)\ C_3^{32}a_3b_3$ $=818\,C_3^{32}a_3b_3$ ways. There are 80 rows of 34 balls each in the same layer, 184 occupying two layers and 96 occupying three layers.

Hence such a row may be formed in, $(80+4.184+6.96)C_4^{33}a_4b_4 =1392C_4^{33}a_4b_4$ ways.

In a similar manner we find,

For 35 balls, $(50+5.132+10.148)C_5^{34}a_5b_5=219C_5^{34}a_5b_5$ ways.

For 36 balls, $(40\times 6.80+15.104+20.56)C_6^{35}a_6b_6=3200C_6^{35}a_6b_6$ ways.

For 37 balls, $(\ 0+7.64+21.60+35.76)C_7^{36}a_7b_7=4438C_7^{56}a_7b_7$ ways.

For 38 balls, $(20+8.48+28.48+56.40+70.24)C_8^{37}a_8b_8=5668C_8^{37}a_8b_8$ ways.

For 39 balls, $(10+9.32+36.36+84.32+126.20)C_9^{38}a_9b_9=6802C_9^{38}a_9b_9$ ways.

For 40 balls, $(10.16+45.24+120.24+210.16)C_{10}^{39}a_{10}b_{10}=7480C_{10}^{39}a_{10}b_{10}$ways.

For 41 balls, $(55.12+165.16+330.12)C_{11}^{40}a_{11}b_{11}=7260C_{11}^{40}a_{11}b_{11}$ ways.

For 42 balls, $(220.8+495.8)C_{12}^{41}a_{12}b_{12}=5720C_{12}^{41}a_{12}b_{12}$ ways.

For 43 balls, $715.4C_{13}^{42}a_{13}b_{13}=2860C_{13}^{42}a_{13}b_{13}$ ways.

Let n, n_1, $n_2 \ldots . n_{13}$ represent the numbers of ways of forming the different rows as above shown and let N_1 represent the total number of ways of drawing the balls. Thus $N_1=\underline{|1500}$, and the probability sought is,

$$p=\frac{n+n_1+n_2+\ldots . n_{13}}{N_1}.$$

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

ANSWER TO A QUERY IN JUNE MONTHLY.

On page 214 of the *American Mathematical Monthly*, Vol. I. No. 6., appears a Query in regard to the work of Giordano da Bitonto: Euclide restituto overe gli antichi elementi geometrici ristoranti; Roma, 1680. Folio.

In answer to this Dr. Halsted has received a letter from the celebrated Gino Loria of the University of Genoa, of which the following is a translation:

GENOA, AUGUST 27th 1894.

HONORED COLLEAGUE,

Having read the question which you have proposed at p. 214 of the *American Mathematical Monthly*, I have asked Mr. Ricardi, former professor of the University of Modena, to give a reply to it. He has answered that the work of Giordano di Bitonto which interests you, and even a falsification which appeared in 1681 exist in his own rich private library. Both are described in the great work of this professor intitled *Biblioteo matematici italiana*, of which a second edition is now in press I hope that these data will suffice you. If not, address me, and I will be glad to furnish the information you desire.

I await the continuation of your interesting article on the non-Euclidean geometry.

Accept, dear sir, the expression of my distinguished consideration.

Your very devoted,
GINO LORIA.